

Subject.

Date.

Sh(1)

$$\text{Max: } Z = 2x_1 + 3x_2$$

$$Z - 2x_1 - 3x_2 + 0S_1 + 0S_2 = 0$$

$$\text{s.t. } 2x_1 + x_2 \leq 4$$

$$\Rightarrow 2x_1 + x_2 + S_1 = 4$$

$$x_1 + 2x_2 \leq 5$$

$$x_1 + 2x_2 + S_2 = 5$$

$$x_1, x_2 \geq 0$$

Basic	Z	x_1	x_2	S_1	S_2	Sol	ratio
Z	1	-2	-3	0	0	0	
S_1	0	2	1	1	0	4	4
S_2	0	1	2	0	1	5	2.5

Ratio

	1	$-\frac{1}{2}$	0	0	$-\frac{3}{2}$	$\frac{15}{2}$	
S_1	0	$\frac{3}{2}$	0	1	$-\frac{1}{2}$	$\frac{3}{2}$	1
x_2	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{5}{2}$	5

$$S_1 = \frac{3}{2}, S_2 = 0, x_1 = 0, x_2 = \frac{5}{2} \Rightarrow Z = \frac{15}{2}$$

	1	0	0	$\frac{1}{3}$	$\frac{4}{3}$	8	
x_1	0	1	0	$\frac{2}{3}$	$-\frac{1}{3}$	1	$x_1 = 1, x_2 = 2, S_1 = S_2 = 0 \Rightarrow Z = 8$
x_2	0	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	2	

Sh(2)

$$\text{Min: } Z = 4x_1 + x_2$$

$$Z - 4x_1 - x_2 + 0S_1 + 0S_2 + 0S_3 - MR_1 + MR_2 = 0$$

$$\text{s.t. } 3x_1 + x_2 = 3$$

std form

$$3x_1 + x_2 + R_1 = 3$$

$$4x_1 + 3x_2 \geq 6$$

 $\Rightarrow$

$$4x_1 + 3x_2 - S_2 + R_2 = 6$$

currentity into

$$x_1 + 2x_2 \leq 4$$

$$x_1 + 2x_2 + S_3 = 4$$

$$x_1, x_2 \geq 0$$

$$x_1, x_2, S_1, S_2, S_3, R_1, R_2$$

Basic	Z	x_1	x_2	S_2	S_3	R_1	R_2	Sol.	zero $\Rightarrow$ no effect
Z	1	-4	-1	0	0	-M	-M	0	
R_1	0	3	1	0	0	1	0	3	
R_2	0	4	3	-1	0	0	1	6	
S_3	0	1	2	0	1	0	0	4	

to remove
 $R_1 \rightarrow R_1$
 $R_2 \rightarrow R_2$

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Basic	Z	x_1	x_2	S_2	S_3	R_1	R_2	Sol.	Ratio
Z	1	$4+7M$	$-1+4M$	$-M$	0	0	0	$9M$	
R_1	0	3	1	0	0	1	0	3	1
R_2	0	4	3	-1	0	0	1	6	$\frac{3}{2}$
S_3	0	1	2	0	1	0	0	4	4

$$x_1=0, x_2=0, S_2=0, S_3=4, R_1=3, R_2=6 \Rightarrow Z=9M$$

Basic	Z	x_1	x_2	S_2	S_3	R_1	R_2	Sol.	Ratio
Z	1	0	$\frac{1}{3}+\frac{5}{3}M$	$-M$	0	$\frac{4-7}{3}M$	0	$4+2M$	
x_1	0	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	1	3
R_2	0	0	$\frac{2}{3}$	-1	0	$-\frac{4}{3}$	1	2	1.2
S_3	0	0	$\frac{3}{3}$	0	1	$-\frac{1}{3}$	0	3	1.8

$$x_1=1, x_2=0$$

$$S_2=0, S_3=3$$

$$R_1=0, R_2=2$$

$$\Rightarrow Z=4+2M$$

Basic	Z	x_1	x_2	S_2	S_3	R_1	R_2	Sol.	Ratio
Z	1	0	0	$\frac{1}{3}M$	0	$\frac{8}{3}-M$	$-\frac{1}{3}M$	$\frac{10}{3}$	
x_1	0	1	0	$\frac{1}{3}$	0	$\frac{3}{3}$	$-\frac{1}{3}$	$\frac{3}{3}$	3
x_2	0	0	1	$-\frac{3}{3}$	0	$-\frac{4}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	-ve
S_3	0	0	0	1	1	1	-1	1	1

$$x_1=0.6, x_2=1.2, S_2=0, S_3=1, R_1=R_2=0 \Rightarrow Z=15.8$$

Basic	Z	x_1	x_2	S_2	S_3	R_1	R_2	Sol.	Ratio
Z	1	0	0	0	$-\frac{1}{5}$	$\frac{7}{5}-M$	$-M$	$\frac{17}{5}$	
x_1	0	1	0	0	$\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{2}{5}$	
x_2	0	0	1	0	$\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{9}{5}$	
S_2	0	0	0	1	1	1	-1	1	1

No positive value $\Rightarrow$ optimum solution

$$x_1=0.4, x_2=1.8, S_2=1, S_3=0, R_1=R_2=0 \Rightarrow Z=5.6$$

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Sh(3)

Basic	Z	x_1	x_2	S_1	S_2	S_3	Solo ratio
Z	1	-3	-5	0	0	0	0
S_1	0	1	0	1	0	0	4
S_2	0	0	(2)	0	1	0	12
S_3	0	3	1	0	0	1	18

2-Z	1	+3	0	5/2	0	0	30
S_1	0	1	0	1/1	0	0	4
x_2	0	0	1	0	1/2	0	6
S_3	0	(3)	0	-10	1/2	0	6

3-	1	0	0	12/5	0	1/3	36/5
S_1	0	0	0	30/5	1	+1/6	2
x_2	0	0	1	-0/5	1/2	0	1/6
x_1	0	1	0	0/5	1/6	0	3/25

$$x_1 = 2, x_2 = 6 \Rightarrow Z = 36$$

Sh(4)

Basic	Z	x_1	x_2	S_1	S_2	R_3	S_0
Z	1	-3	-5	0	0	M	0
S_1	0	1	0	1	0	0	4
S_2	0	0	2	0	1	0	12
R_3	0	3	2	0	0	1	18

								Ratio
	1	-3-3M	-5-2M	0	0	0	-18M	
S_1	0	(1)	0	1	0	0	4	4
S_2	0	0	2	0	1	0	12	6
R_3	0	3	2	0	0	1	18	6

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$$\begin{array}{cccc|cccc}
 1 & -0.3 & 0.5 & -2 & 0 & 0 & 12 & 6 \\
 x_1 & 0 & 1 & 0 & 1 & 0 & 0 & 4 \\
 s_2 & 0 & 0 & 2 & 0 & 1 & 0 & 12 \\
 R_3 & 0 & 0 & 2 & -3 & 0 & 1 & 6
 \end{array}$$

$$\begin{array}{cccc|cccc}
 1 & 0 & 0 & 1 & 9/2 & 0 & 5/2 & 27 \\
 x_1 & 0 & 1 & 0 & 1 & 0 & 0 & 4 \\
 s_2 & 0 & 0 & 0 & -3 & 1 & -2 & 6 \\
 x_2 & 0 & 0 & 1 & -3/2 & 0 & 1/2 & 3
 \end{array}$$

$$\begin{array}{cccc|cccc}
 1 & 0 & 0 & 0 & 3/4 & 1 & M & 36 \\
 x_1 & 0 & 1 & 0 & 0 & -1/3 & 1/3 & 2 \\
 s_1 & 0 & 0 & 0 & 1 & 1/3 & -1/3 & 2 \\
 x_2 & 0 & 0 & 1 & 0 & 1/2 & 0 & 6
 \end{array}$$

optimum solution : $x_1 = 2, x_2 = 6, s_1 = 2, s_2 = 0, R_3 = 0$

Sh(s)

Min : $Z = 0.4x_1 + 0.5x_2$

s.t. : $0.3x_1 + 0.1x_2 \leq 2.7$

$0.5x_1 + 0.5x_2 = 6$

$0.6x_1 + 0.4x_2 \geq 6$

$x_1, x_2 \geq 0$

$Z = 0.4x_1 + 0.5x_2 + 0s_1 + 0s_2 - MR_2 - MR_3 = 0$

$0.3x_1 + 0.1x_2 + s_1 = 2.7$

$0.5x_1 + 0.5x_2 + R_2 = 6$

$0.6x_1 + 0.4x_2 + s_3 + R_3 = 6$

$x_1, x_2, s_1, s_3, R_2, R_3 \geq 0$

Basic	Z	x_1	x_2	s_1	s_3	R_2	R_3	Sol.
Z	1	$-\frac{2}{5}$	$-\frac{1}{2}$	0	0	-M	-M	0
s_1	0	$\frac{3}{10}$	$\frac{1}{10}$	1	0	0	0	$\frac{27}{10}$
R_2	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0	6
R_3	0	$\frac{3}{5}$	$\frac{2}{5}$	0	-1	0	1	6

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Sh(6)

Z	x_1	x_2	S_1	S_2	Sol	Ratio	Sol
Z	1	-3	-9	0	0		
S_1	0	1	4	1	0	2	1
S_2	0	1	2	0	1	2	3/5

Degereneration

$$x_1=0, x_2=0, S_1=2, S_2=2 \Rightarrow Z=0$$

Z	x_1	x_2	S_1	S_2	Sol	Ratio
Z	N/A	3/2	0	0	9/2	18
S_1	0	-1	0	1	-2	0
S_2	0	1/2	1	0	1/2	2

no +ve values $\Rightarrow$ optimal solution

$$x_1=0, x_2=2, S_1=0, S_2=0 \Rightarrow Z_{opt}=18$$

Sh(7)

Z	x_1	x_2	S_1	S_2	Sol	Ratio
Z	1	-2	-4	0	0	
S_1	0	1	2	1	0	5
S_2	0	1	1	0	1	4

$$x_1=0, x_2=0, S_1=5, S_2=4 \Rightarrow Z=0$$

Z	x_1	x_2	S_1	S_2	Sol	Ratio
Z	1	0	0	2	0	10
x_2	0	1/2	1	1/2	0	5/2
S_2	0	1/2	0	-1/2	1	3/2

No positive values but 0, 0 at basic variables $\Rightarrow$ solution has optimal alternatives

Z	x_1	x_2	S_1	S_2	Sol
Z	1	0	0	2	0
x_2	0	0	1	1	-1
x_1	0	1	0	-1	2

$$x_1 + 2x_2 \leq 5$$

The optimal solution alternative lie on line starts from (0, 2.5) to (2.5, 0)

$$x_2 = 2.5 - \frac{1}{2}x_1 \Rightarrow 4x_1 + 2x_2 = 5$$

Sh(8):

10/12

Z	x_1	x_2	S_1	S_2	R_2	R_2 Sol.	Ratio
Z	1	-3	-4	2	0	M	0
S_1	0	12	4	1	0	0	225
R_2	0	13	4	0	-1	1	12

$$x_1 = x_2 = 0, S_1 = 5, S_2 = 9 \Rightarrow Z = -12M$$

Z	x_1	x_2	S_1	S_2	R_2	Sol.	Ratio
Z	1	-3-3M	-24M	0	M	0	-12M
S_1	0	2/2	(1)	1/2	0	2	2
R_2	0	3	4	0	-1	1	12

$$x_1 = x_2 = 0, S_1 = 2, S_2 = 0, R_2 = 12 \Rightarrow Z = -12M$$

Z	x_1	x_2	S_1	S_2	R_2	Sol.	Ratio
Z	1	1+3M	0	2+4M	M	0	4-4M
x_2	0	2	1	1	0	2	
R_2	0	-5	0	-4	-1	1	4

All values are positive & solution has artificial variable
 $\Rightarrow$ No feasible solution.

Z	1	0	3/4	3/4	-1/2	15/2
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Sh(9)

1	1/2	1/2	0	0	1
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R_2	Z	x_1	x_2	S_1	S_2	Sol.	Ratio
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Z	1	-2	-1	0	0	0	
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S_1	0	(1)	-1	1	0	10	10
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S_2	0	2	0	0	1	40	20
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$$x_1 = x_2 = 0, S_1 = 10, S_2 = 40 \Rightarrow Z = 0$$

Z	x_1	x_2	S_1	S_2	Sol.	Ratio
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Z	1	0	-3	2	0	20
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x_1	0	1	-1	1	0	10	-ve
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S_2	0	0	(2)	-2	1	20	10
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$$x_1 = 10, x_2 = 0, S_1 = 0, S_2 = 20 \Rightarrow Z = 20$$

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Z	x_1	x_2	S_1	S_2	Sol	Ratio
Z	1	0	0	-1	$3/2$	50
x_1	0	1	0	0	$1/2$	20
x_2	0	0	1	-1	$1/2$	10

$$x_1 = 20, x_2 = 10, S_1 = S_2 = 0 \Rightarrow Z = 50$$

Ratios aren't positive $\Rightarrow$ Solution is unbounded